

A Gravitational Arrow of Time and New Entropy Concept

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Collaborators and References

Tim Koslowski, Flavio Mercati and David Sloan

JB, Tim Koslowski and Flavio Mercati “Identification of a gravitational arrow of time”, Phys. Rev. Lett. **113**, 181101 (2014).

JB, Tim Koslowski and Flavio Mercati “Entropy and the typicality of universes”, arXiv: 1507.06498v2.

Tim Koslowski, Flavio Mercati and David Sloan “Through the Big Bang: Continuing Einstein’s equations beyond a cosmological singularity”
Physics Letters B **778**, 339-343 (2108).

Diagrams by Flavio Mercati

This book is about the nature of space and time. It is the first textbook on Shape Dynamics, a new theory which describes the dynamics of gravity as the evolution of conformal 3-dimensional geometry. Shape Dynamics is equivalent to Einstein's General Relativity in those situations in which the latter has been tested experimentally, but the theory is based on different first principles. It differs from General Relativity in certain extreme conditions. Shape Dynamics allows us to describe situations in which the spacetime picture is no longer adequate, such as in the presence of singularities, when the idealization of infinitesimal rods measuring scales and infinitesimal clocks measuring proper time fails.

This tutorial book contains both an introduction for readers curious about Shape Dynamics, and a detailed walk-through of the historical and conceptual motivations for the theory, its logical development from first principles and a description of its present status. It includes an explanation of the origin of the theory, starting from problems posed first by Newton more than 300 years ago. The book will interest scientists from a large community including all foundational fields of physics, from quantum gravity to cosmology and quantum foundations, as well as researchers interested in foundations. The tutorial is sufficiently self-contained for students with some basic background in Lagrangian/Hamiltonian mechanics and General Relativity.

FLAVIO MERCATI is a postdoctoral researcher at the Perimeter Institute for Theoretical Physics. In 2015 he won the Buchalter Prize for Cosmology for his research on the arrow of time.

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SHAPE DYNAMICS

RELATIVITY AND
RELATIONALISM

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SHAPE DYNAMICS

RELATIVITY AND RELATIONALISM

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Overview

- Scale as Distinguished Degree of Freedom

Confined and Unconfined Systems

- Carnot's Benign and Malign Influence

Why Boltzmann Failed: The Real Problem of Time's Arrows

- Unconfined Janus-Point Systems

Illustrated by 3- and N -Body Problems

- Shape Space and Scale-Invariant Complexity

Origin of All Arrows of Time

- Entropy-Like Quantity for Universe Decreases

Emergence of Second Law

- Order and Disorder

Confined and Unconfined Systems

- The Final State of the Universe

Heterogeneity of Degrees of Freedom

An N -particle system has:

3 centre-of-mass DoFs

3 orientational DoFs

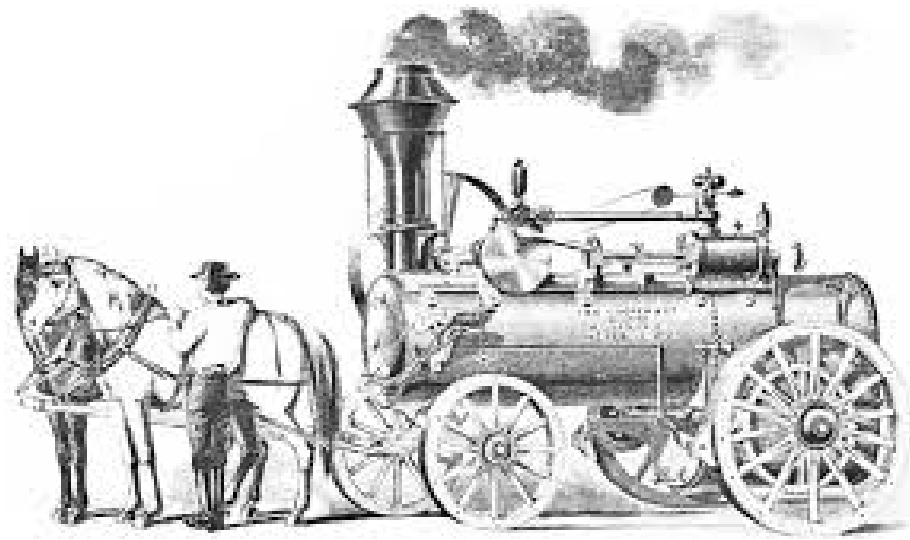
1 scale DoF

$3N - 7$ shape DoFs

In GR, 1 scale DoF, infinitely many shape DoFs

Observable shape DoFs behave very differently if
scale DoF bounded or unbounded

Sadi Carnot and Steam Engines



Reflections on the Motive Power of Heat (1824)

Steam is confined to a cylinder. Virtually all experimental and theoretical work has presupposed a confining 'box'.

Comments on Carnot

Showed how to use coal to maximum efficiency.

But what is the origin of coal?

“The production of heat alone is not sufficient to give birth to impelling power: it is necessary that there should also be cold; without it, the heat would be useless. And in fact, if we should find around us only bodies as hot as our furnaces, how could we condense steam?”

What is the origin of heat differences?

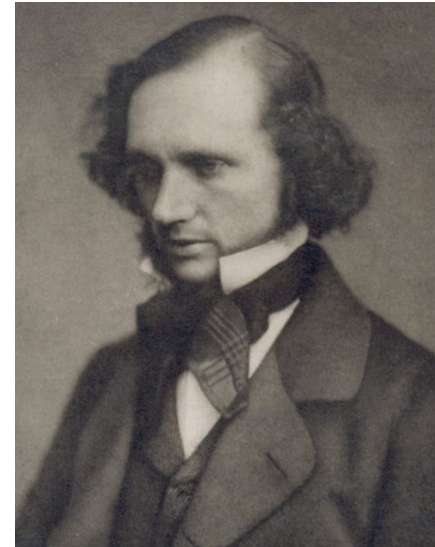
Anthropocentricity: steam engines do work for humans

Physical Reality: they change the shape of the universe

Time-Reversal Symmetry

The laws of nature distinguish no direction of time.

Whence comes the **arrow of time**? A mystery since 1852: *On a universal tendency in nature to the dissipation of mechanical energy* (William Thomson, aka Lord Kelvin). “One of the gloomiest scientific predictions of all time.” NB: Dissipation does not mean destruction but making less concentrated.



“One need look no further than the Sun, slowly burning through its stock of nuclear fuel, radiating heat and light irreversibly into the cold depths of space, to see an infinitesimal contribution to the heat death.” Paul Davies (2013).

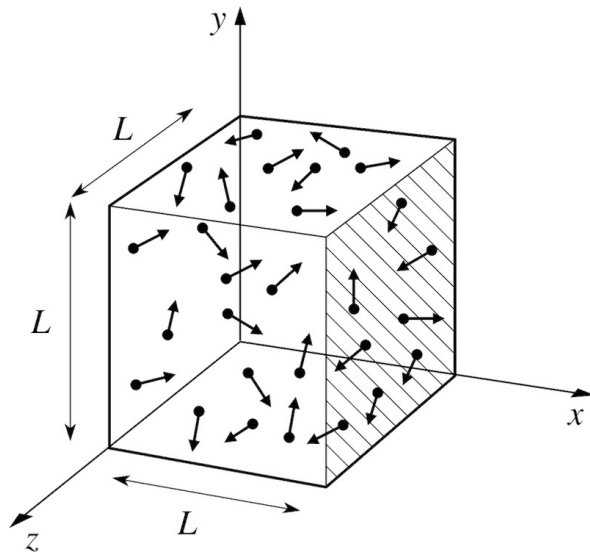
Comments on Kelvin

Introduced concept of **stores of mechanical energy**, which could be statical or dynamic. Accepted Helmholtz's nebular origin of sun and planets but had **no theory of origin of the nebula**.

“It is impossible to conceive a limit to the extent of matter in the universe; and therefore science points rather to an endless progress, through an endless space, of action involving the transformation of potential energy into palpable motion and thence into heat, than to a single finite mechanism, running down like a clock, and stopping for ever.” (Kelvin, 1862)

Significance of Box

Box allows equilibration from a non-equilibrium state.



Controlled heat flow. Measurement of P , V , T in equilibrium. Clausius's definition of entropy increment:

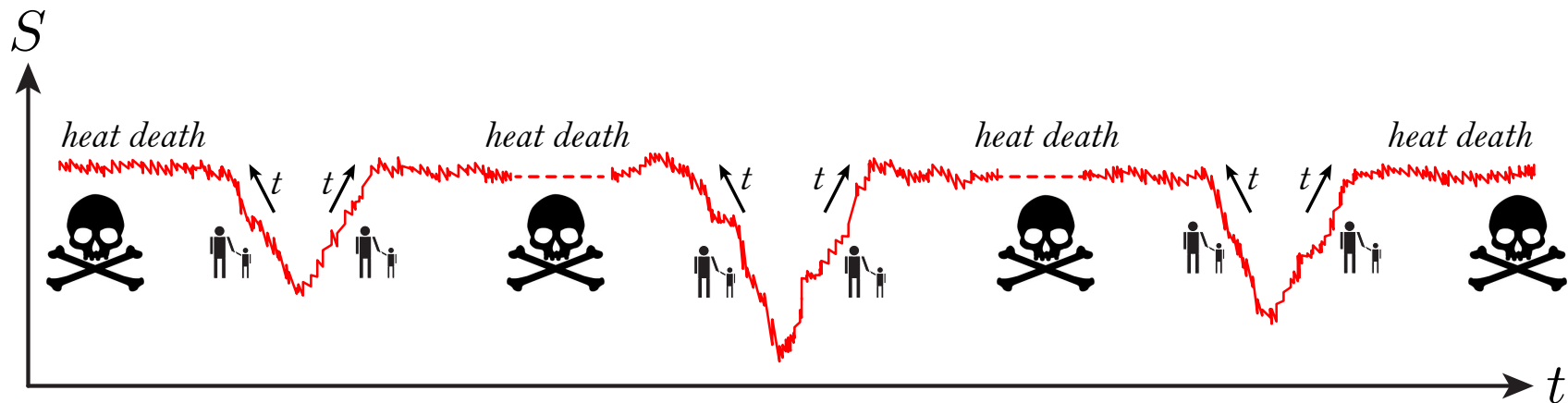
$$dS = \frac{dQ}{T_{\text{abs}}}$$

Gibbs (1902): General Hamiltonian dynamics in **phase space of bounded measure** in order to define probabilities.

No real progress on time's arrows.

Consequences of Confining Box

Expansion 'thwarted' \Rightarrow time-symmetric Poincaré recurrence



Boltzmann (1895): “The universe is, and rests forever, in thermal equilibrium.” Only near deep entropy dips “are worlds where visible motion and life exist . . . the direction of time towards the more improbable state [will be experienced as] the past.”

Key insight: Time has no pre-existing direction.

Each dip has one past but two futures.

The Real Problem

Boltzmann in response to Zermelo in 1996:

“Naturally, we cannot expect from natural science an answer to the question – how does it happen that at present the bodies surrounding us are in a very improbable state?”

The aim of this talk: to find the answer.

“If we should find around us only bodies as hot as our furnaces, how could we condense steam?” (Carnot, 1824).

And a first-principles dynamical explanation why hot and cold exist and steam can be condensed.

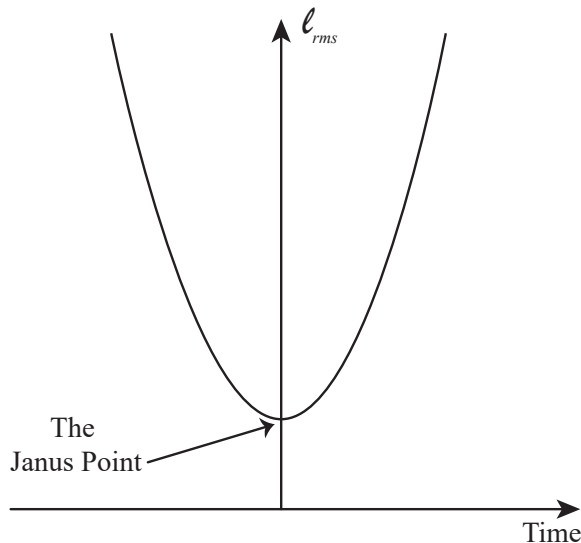
Lagrange's 1772 Qualitative Result

$$I_{\text{cm}} = \sum_{a=1}^{a=N} m_a \mathbf{r}_a^{\text{cm}} \cdot \mathbf{r}_a^{\text{cm}} \equiv m_{\text{tot}} \sum_{a < b} \frac{m_a m_b r_{ab}^2}{m_{\text{tot}}^2} = m_{\text{tot}} \ell_{\text{rms}}^2.$$

I_{cm} : centre-of-mass moment of inertia, ℓ_{rms} : rms length

If V homogeneous, $V(\alpha \mathbf{r}_a) = \alpha^k V(\mathbf{r}_a)$, then $\frac{1}{2} \ddot{I}_{\text{cm}} = E_{\text{cm}} - 2(k+2)V$.

If $E_{\text{cm}} \geq 0$, then because $V_{\text{New}} < 0$ and $k = -1$ we have $\ddot{I}_{\text{cm}} > 0$



I_{cm} is U-shaped upwards, the dilational momentum $D = \sum_a \mathbf{r}_a \times \mathbf{p}_a = \frac{1}{2} \dot{I}_{\text{cm}}$ is monotonic and vanishes once at the **Janus point**, the minimum of ℓ_{rms} , which divides every solution with $E_{\text{cm}} \geq 0$ in half. No Poincaré recurrence. **Not a bounce.**

Janus-Point Systems



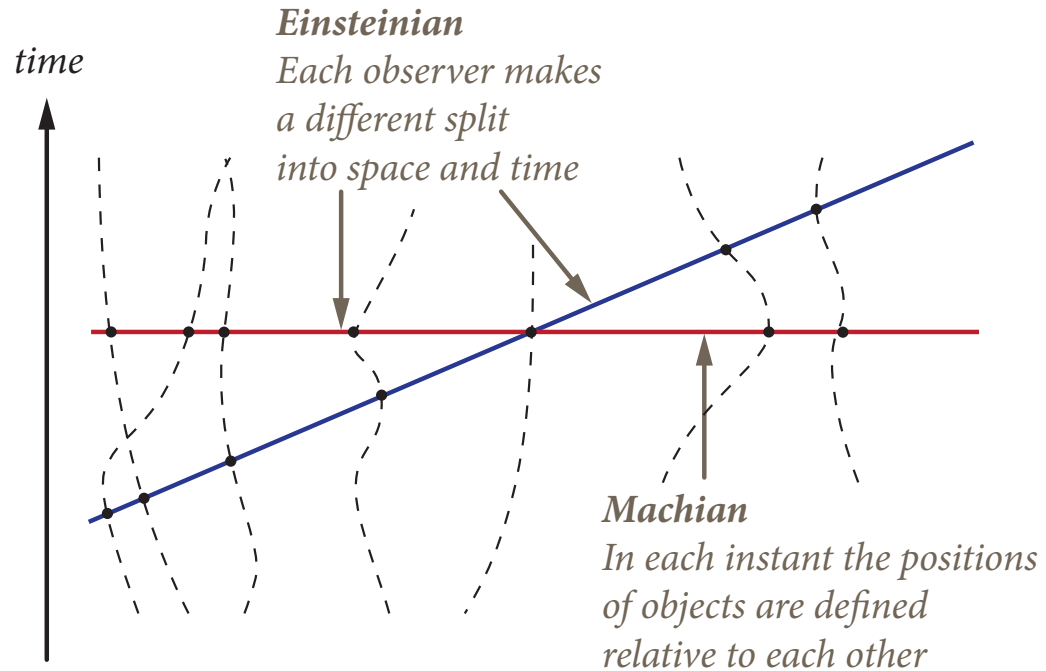
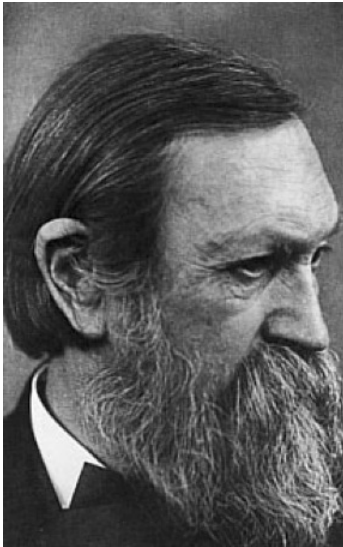
Every solution divides at unique Janus point **J**.

Evolution **time-asymmetric** either side of **J**.

Arrows of time point away from **J** in each half.

Hypothesis: The Law of the Universe dictates
a Janus point in all of its solutions.

Instants and Two Relativities



“The universe is given once only, with its
relative motions alone determinable.”

An instant: the shape of a finite relative configuration.”

Shape Space S

‘Expansion of universe’ is really evolution of ratios:

$$\frac{\text{Galactic Diameters}}{\text{Inter-Galactic Separations}} \rightarrow 0$$

Dimensionless ratios alone have physical meaning.

S: the space of the Universe’s possible shapes

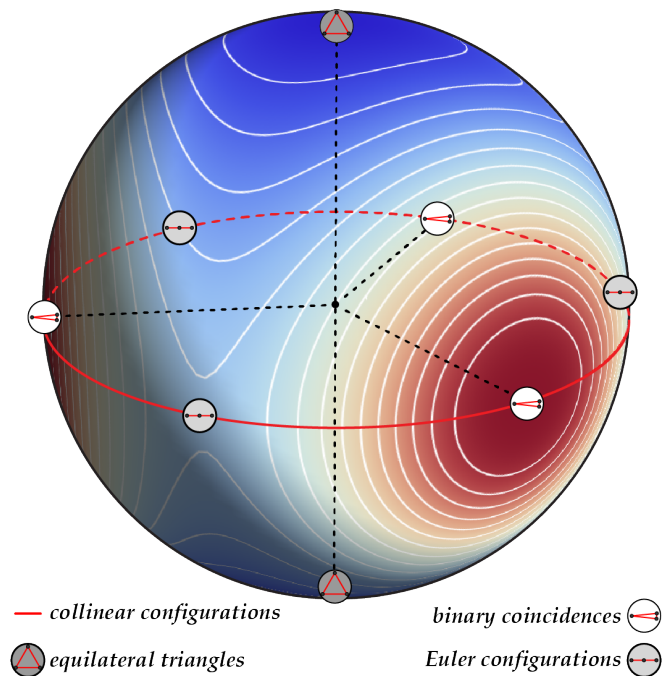
3-Body Shape Sphere

S^3 : the space of triangle shapes

Plotted for equal masses

Universe ‘boxes’ itself

Colour coding next slide.



A Scale-Invariant Measure of Clustering

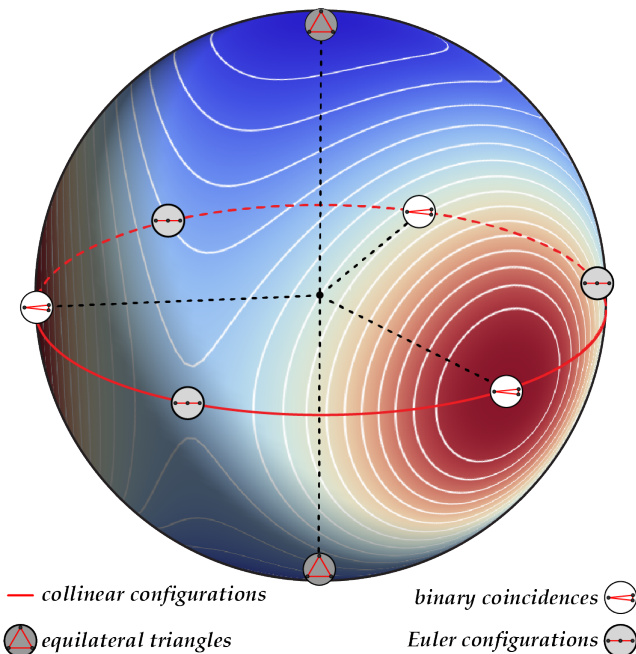
Ratio of **root-mean-square length** ℓ_{rms}
and **mean harmonic length** ℓ_{mhl}

$$\ell_{\text{rms}} := \sqrt{\sum_{a < b} \frac{m_a m_b r_{ab}^2}{m_{\text{tot}}^2}} = \sqrt{I_{\text{cm}}/m_{\text{tot}}}$$

$$\ell_{\text{mhl}}^{-1} = \frac{1}{m_{\text{tot}}^2} \sum_{a < b} \frac{m_a m_b}{r_{ab}} = -\frac{1}{m_{\text{tot}}^2} V_{\text{New}}$$

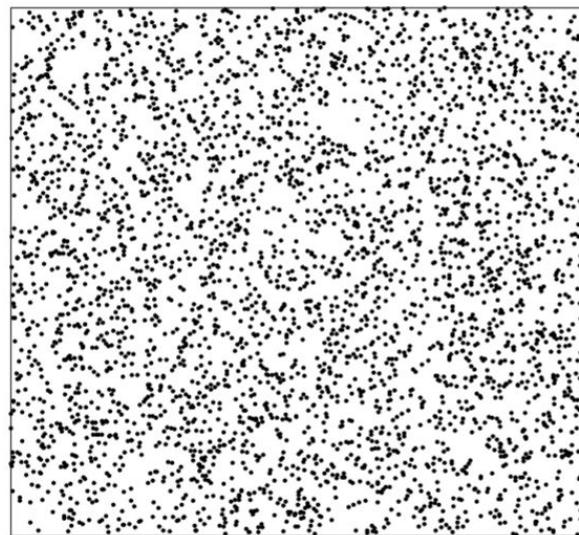
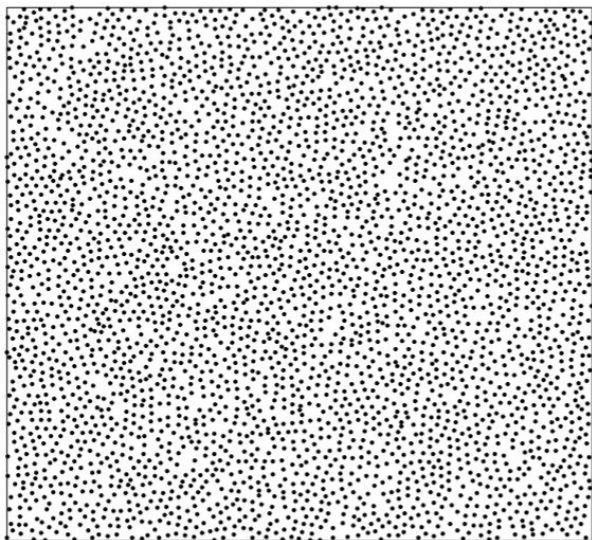
Shape Complexity: $C_{\text{Shape}} = \ell_{\text{rms}}/\ell_{\text{mhl}}$

A sensitive measure of clustering



C_{Shape} is negative of the shape potential.

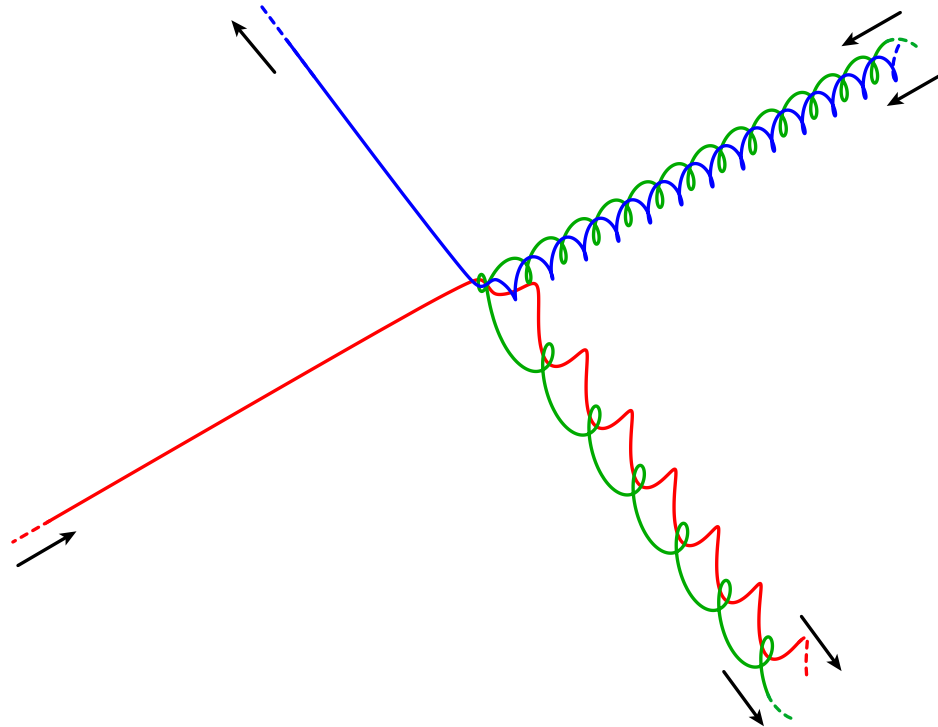
Distinguishing Shapes



The complexity of the glassy distribution (left) is significantly smaller than of the Poisson distribution (right).

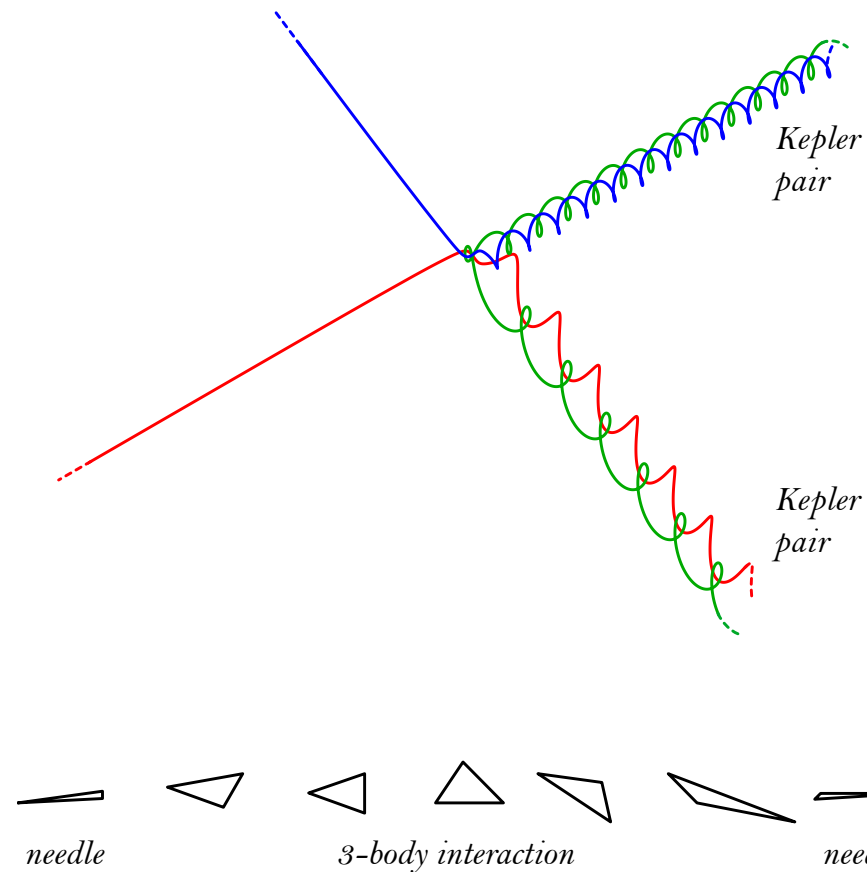
Relational Three-Body Motions

$$E_{\text{cm}} = \mathbf{P} = \mathbf{L} = 0 \text{ (planar because } \mathbf{L} = 0\text{)}.$$



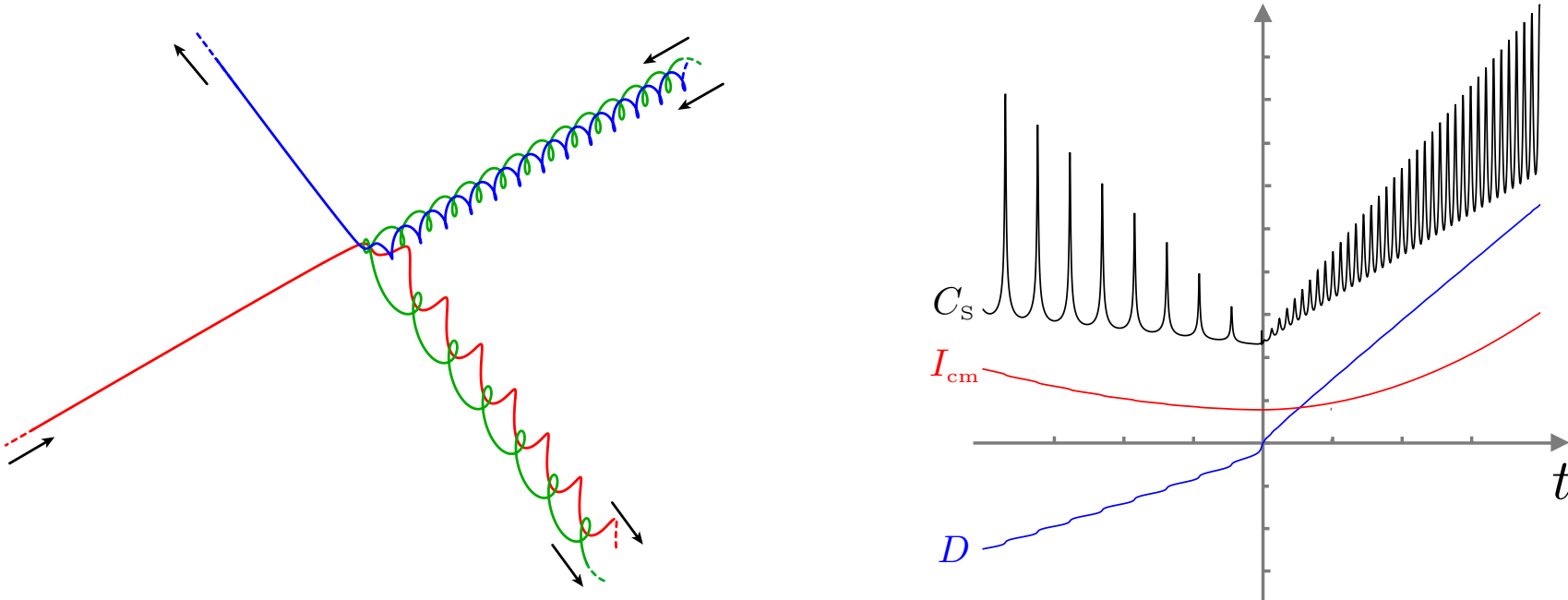
Direction of time for triple-star system fixed by background arrow of the universe but purely nominal for toy universe.

One-Past-Two-Futures



Bidirectional arrows of time in 3-body dynamics. No statistics.
Emergence of rods, clocks, compasses and manifest
inertial frames from chaos.

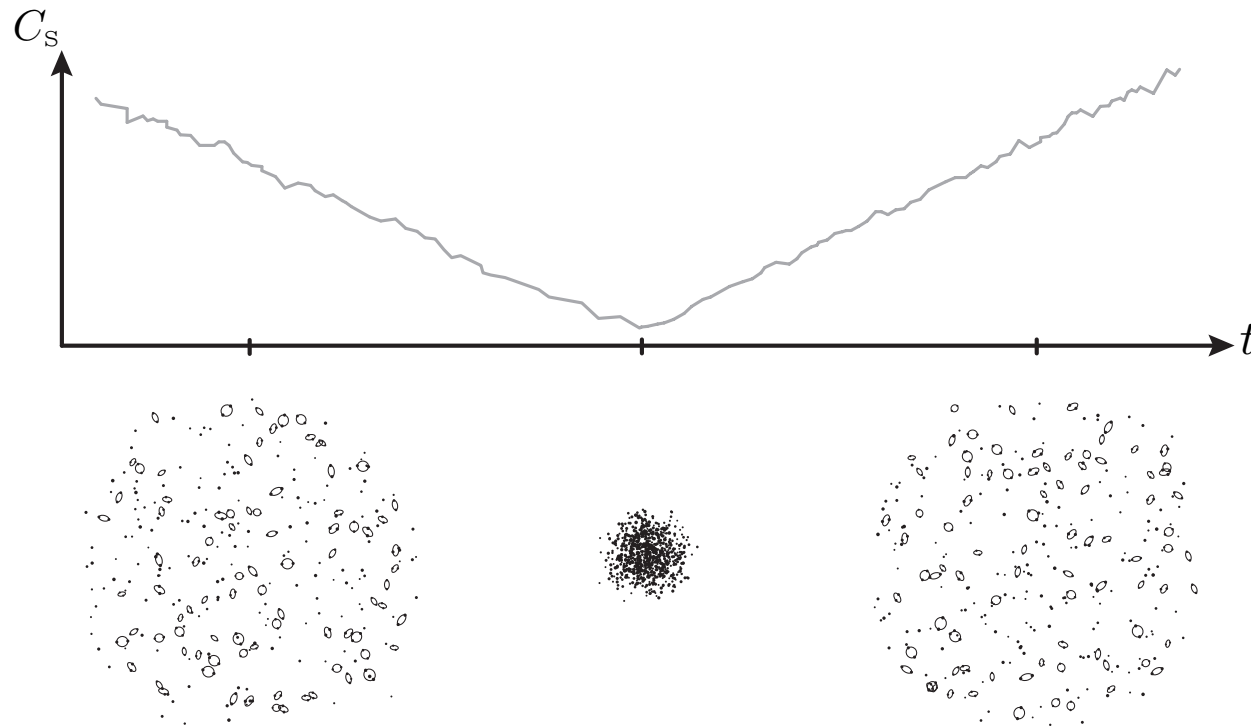
Janus Point 'Seen' in Complexity



The ticks, length and direction defined by the Kepler pair confirm
singleton tends to inertial motion.

The Age of Metrology dawns when orbital elements stabilize.

1000-Body Simulation

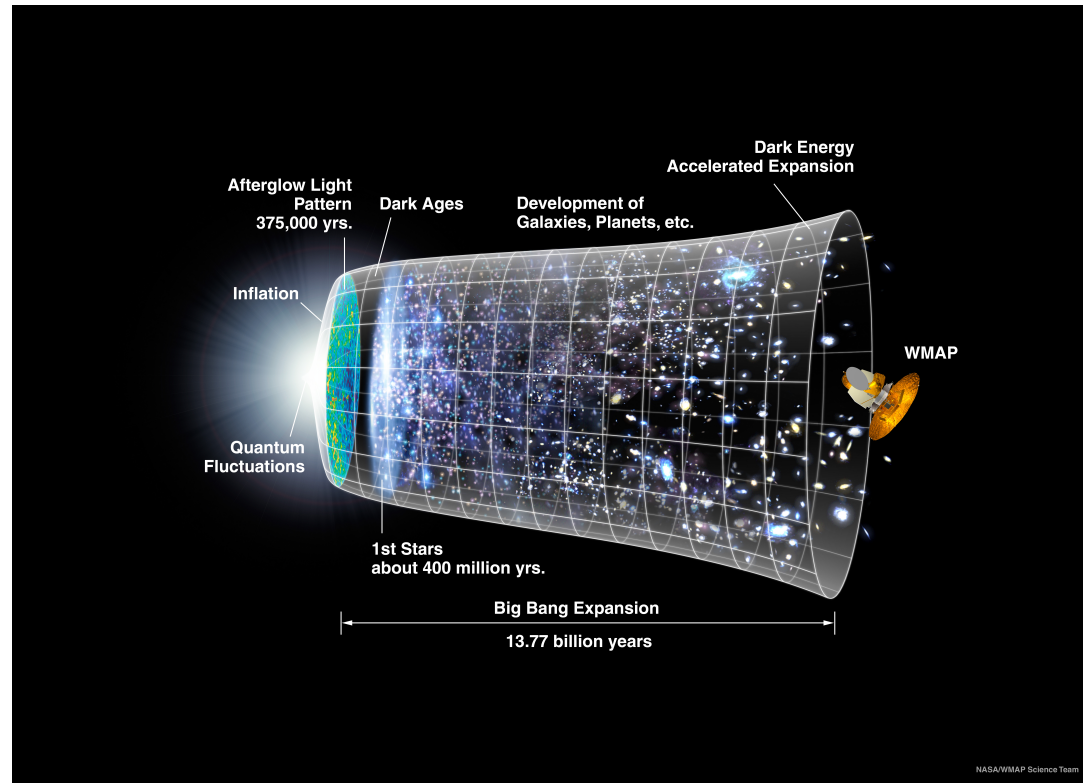


In ‘Artistic Impression’ isolated Kepler pairs form and all march in step remaining mutually congruent with fixed relative directions.

“It is not to be conceived that mere mechanical causes could give birth to so many regular motions . . . This most beautiful system of the sun, planets and comets, could only proceed from the counsel and dominion of an intelligent and powerful Being.”

Isaac Newton

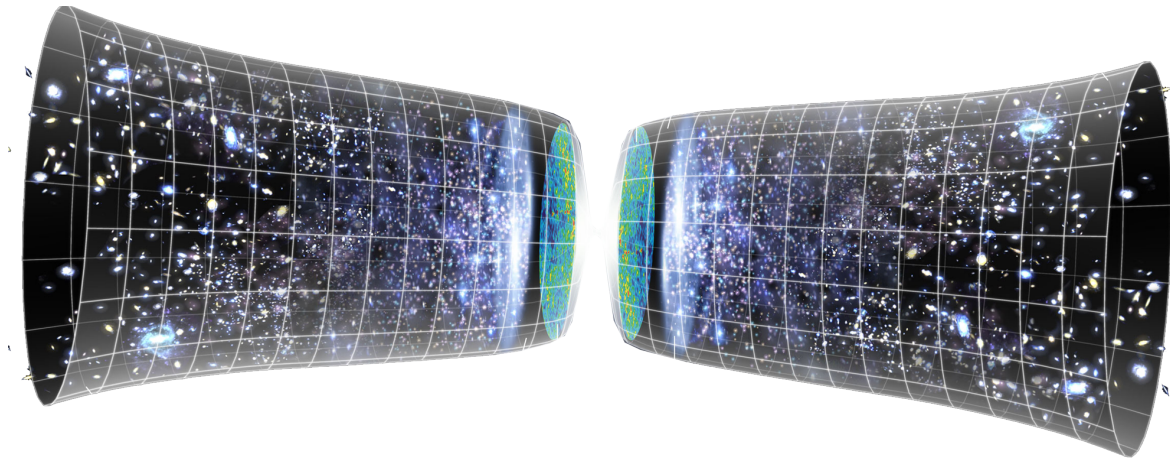
The Story of the Universe According to NASA



Growth of Order, Information and High-Precision Metrology

“Universal tendency in nature to the dissipation of mechanical energy”
or: “Universal tendency of the universe to create structure.”

The Janus-Point Proposal



Volume goes to zero but shape changes smoothly.

Time-reversal symmetry is respected.



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Through the big bang: Continuing Einstein's equations beyond a cosmological singularity



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ABSTRACT

All measurements are comparisons. The only physically accessible degrees of freedom (DOFs) are dimensionless ratios. The objective description of the universe as a whole thus predicts only how these ratios change collectively as one of them is changed. Here we develop a description for classical Bianchi IX cosmology implementing these relational principles. The objective evolution decouples from the volume and its expansion degree of freedom. We use the relational description to investigate both vacuum dominated and quiescent Bianchi IX cosmologies. In the vacuum dominated case the relational dynamical system predicts an infinite amount of change of the relational DOFs, in accordance with the well known chaotic behaviour of Bianchi IX. In the quiescent case the relational dynamical system evolves uniquely though the point where the decoupled scale DOFs predict the big bang/crunch. This is a non-trivial prediction of the relational description; the big bang/crunch is not the end of physics – it is instead a regular point of the relational evolution. Describing our solutions as spacetimes that satisfy Einstein's equations, we find that the relational dynamical system predicts two singular solutions of GR that are connected at the hypersurface of the singularity such that relational DOFs are continuous and the orientation of the spatial frame is inverted.

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Liouville's Theorem and Shape Dynamics

Hamilton's equations

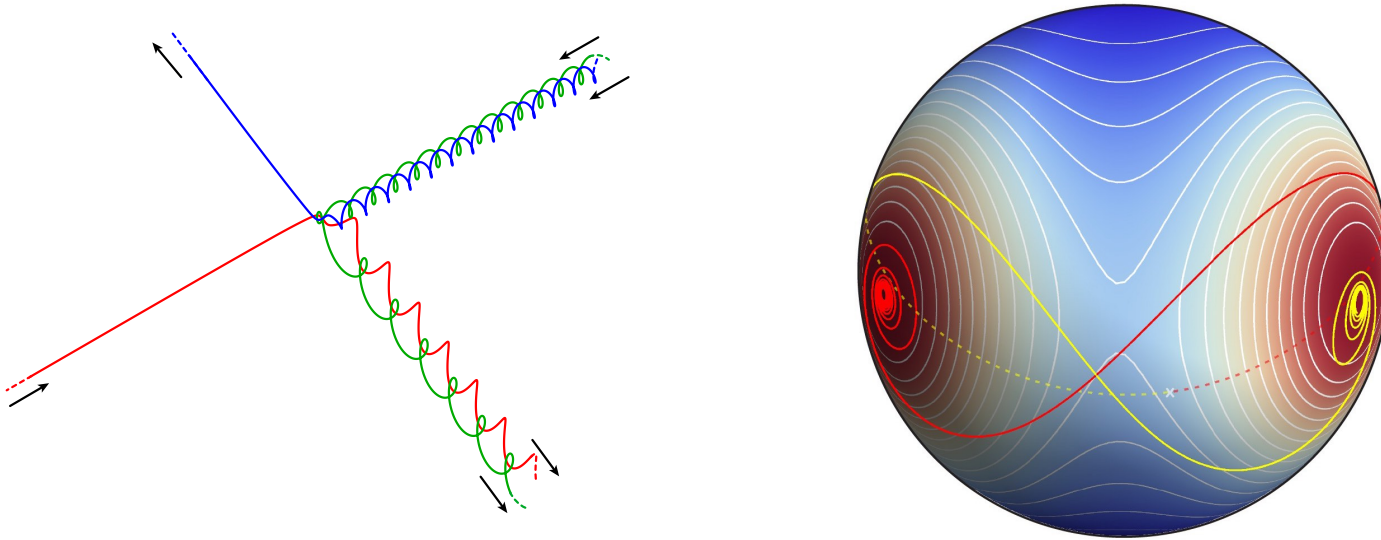
$$\frac{d\mathbf{q}}{dt} = +\frac{\partial \mathcal{H}}{\partial \mathbf{p}}, \quad \frac{d\mathbf{p}}{dt} = -\frac{\partial \mathcal{H}}{\partial \mathbf{q}}$$

conserve phase-space volume

If scale part grows, shape part must decrease \Rightarrow

Attractors on Shape Space

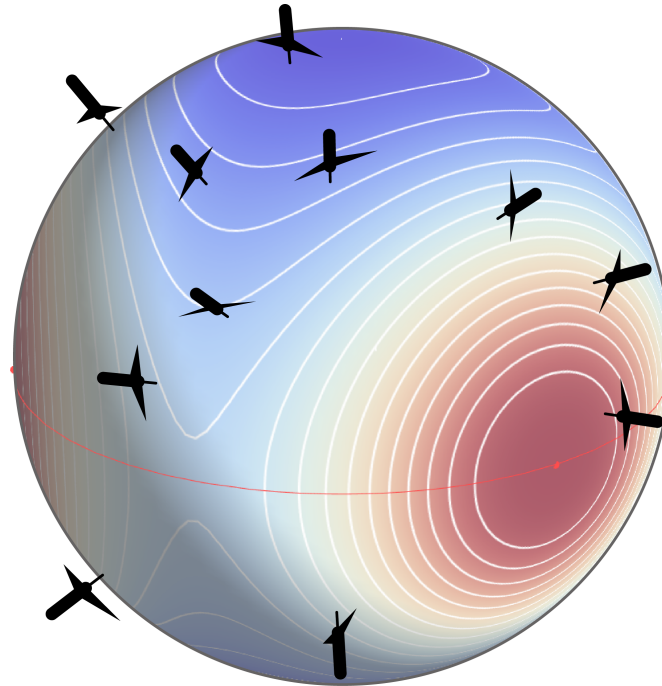
Attractors and the Shape Potential



The ‘attraction’ acts like friction pulling the system into the wells of the shape potential.

Conjecture: all arrows of time due to attractors

Blindfolded Creator Aims for Janus Point

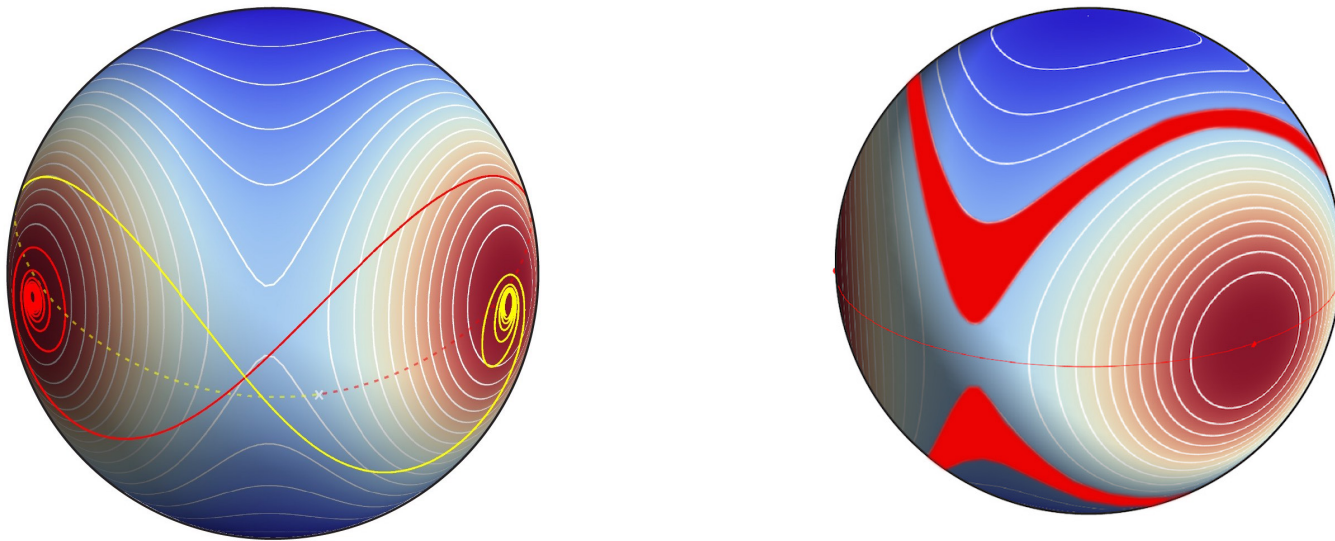


Bulk of darts land in low-complexity region of shape space.
Strongly pronounced for large N .

Our universe began very uniform.

An Entropy-Like Quantity for the Universe

Conventional state functions (energy, volume, etc) fail for the universe.



Use complexity as a state function. **Entaxy** (count of microstates) is area between contours and **tends to decrease** with increasing distance from the Janus point.

Origin of the Second Law

Second Law was discovered in confined systems.

The decrease of the entropy-like entaxy matches formation of self-confined subsystems of the universe in which entropy increases.



Expansion of the universe, gravitational attraction and other forces governed by quantum mechanics \Rightarrow Self-Confinement \Rightarrow Second Law

Final State of Isolated Systems

Tolman (*Relativity, Thermodynamics and Cosmology*, 1934):

If upper limit to possible entropy exists,
this will determine final state of system

For unconfined gases, the final state of **maximum entropy** will be
“infinite dilution and complete dissociation into atoms”

Final state of inertial and N -body systems is **Hubble expansion**.
Does not look like disorder

Entropy of universe increases only if scale a physical DoF

Modern cosmologists take virtually no interest in entropy!

Typical Irreversible Processes

Thermodynamics and Stat Mech: **Highly Atypical Idealizations**

Typical: **Formation** → **Quasi-Equilibration** → **Decay** → **Legacy**



Stars emit spherical-wave 'firework' into cold space.
Black holes fit pattern and don't cause it (Wallace)

Order or Disorder?

Two kinds of unidirectional irreversible processes:

1. In medium with a mean free path p .

Then equilibration within some multiple of p .

2. In dissipationless medium or vacuum.

Then transformation of order (stone in pond, radio waves)
or creation of order: a) structure formation in universe
and b) in and by star.

“The Sun, slowly burning . . . its stock of nuclear fuel,
radiating heat and light irreversibly”

But the Sun is creating all the atoms of which we are made,
and supernovae create the gold with which we decorate ourselves.

Final State of the Universe

Leibniz: “We live in the best of all possible worlds”

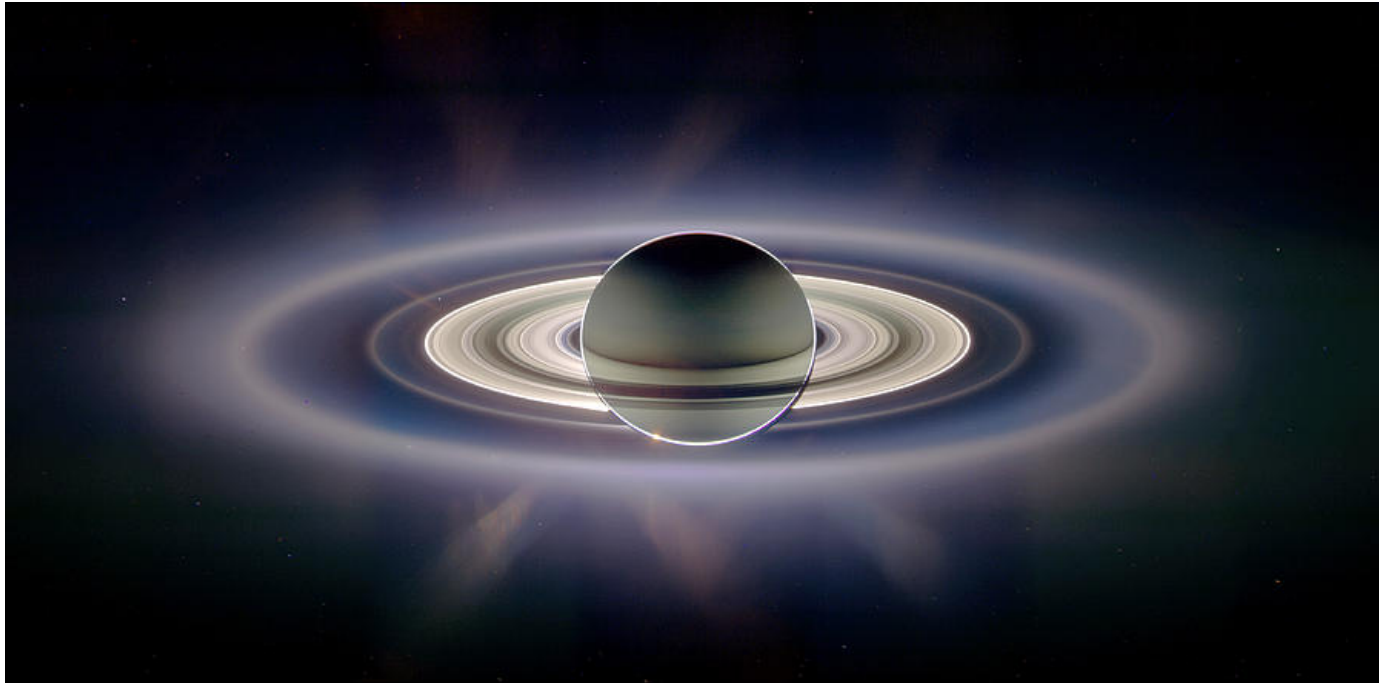
Monadology (1714), Sec. 58. “This is the means of obtaining as much **variety** as possible, but with the **greatest order possible**; that is to say, it is the means of obtaining as much perfection as possible.”

Real numbers can encode an infinite amount of information

**Is infinite Janus-point information unfolded
as fine filigree* in ‘the fullness of time’?**

*Ornamental work of fine (typically gold or silver) wire
formed into delicate tracery

Structure and Beauty Created by Law through Time



Conclusions

1. Confined Systems Create Unidirectional Disorder
2. Unconfined Janus-Point Universes Create Bidirectional Order
3. Unconfined Subsystems Create Unidirectional Order